

## Method of Calculating Optimization of Metal-Rubber Bearing Parameters of Engine by using Gomory Graph

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**Abstract:** Transmission clusters of engine are always the culprit causing oscillation and noise while the car is moving. This component is integrated to the frame of the car by rubber-metal bearing. The oscillation of transmission clusters depends on the force generated when the engine is running and the force stimulated from the frame when the car is moving. The article is a presentation of a method to reduce the oscillation caused by the main transmission cluster by searching for the optimization parameters of the harness of the engine bearing. The exploration of the optimization parameter of the hardness of the transmission cluster bearing is based on building and selection oscillation model, solving differential equations and optimizing the outputs by using Gomory's graph.

**Keywords:** Rubber-Metal, Damping Mount, Optimization, Gomory Graph

### Introduction:

#### The computational model construction:

The engine is integrated to the frame of the car by rubber-metal bearing. We can build dynamic model of engine vibrations on the pillow elastic suspension as Figure 1. This model reflects that the engine has six degrees of freedom in which three degrees of freedom allow the engine relatively shift under the shaft  $ox(X)$ ,  $oy(Y)$ ,  $oz(Z)$ , 3 degrees of freedom allow the motor spin deflection around the shaft  $ox(\varphi)$ ,  $oy(\psi)$ ,  $oz(\theta)$ .

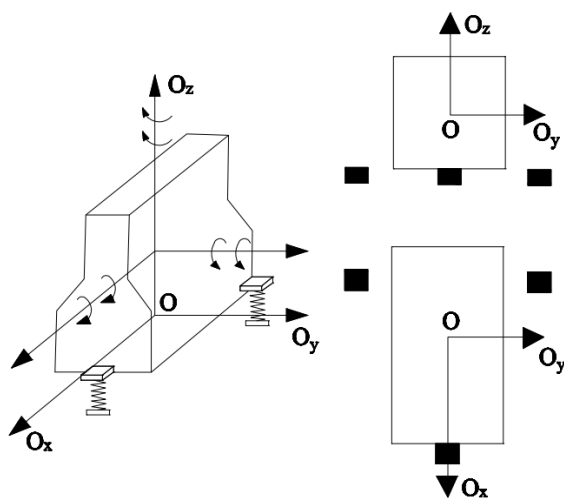


Figure 1: Dynamic model of engine vibrations

When working the engine suffers the major external effects of inertia forces of the moving structure and torque generated at the crankshaft. The vector sum of inertia forces acting on the engine body has the force placement location deviant to the heart of a dynamic cluster at a distance, therefore the main oscillation of the engine cluster is the vertical axis transposition oscillation ( $Oz$ ) and the pendulum oscillation on the horizontal axis of the motor ( $Ox$ ).

Thus in the calculation of the article, when building computational models, the authors have bypassed vertical axis pendulum oscillations of the motor ( $Oy$ ), vertical axis pendulum ( $Oz$ ) and transposition of the  $Ox$  and  $Oy$  axis. Computational model (Figure 2) is referred as a flat model, meaning that the engine has one front bearing and two rear bearings that have hardness value equivalent the value of one bearing.

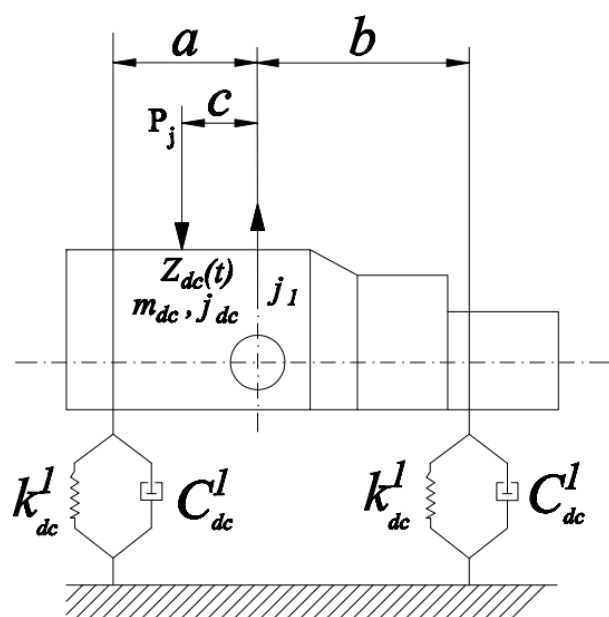


Figure 2: Oscillation model of cluster engine

Oscillating differential equation of the cluster engine is expressed as follows:

$$m_{dc} \ddot{z}_{dc} + (C_{dc}^1 + C_{dc}^2) \dot{z}_{dc} + (k_{dc}^1 + k_{dc}^2) z_{dc} + (C_{dc}^1 b - C_{dc}^2 c) \dot{\varphi}_{dc} + (k_{dc}^1 b - k_{dc}^2 c) \varphi_{dc} = P_{\Sigma};$$

$$J_{dc} \ddot{\varphi}_{dc} + (C_{dc}^1 b^2 + C_{dc}^2 c^2) \dot{\varphi}_{dc} + (k_{dc}^1 b^2 + k_{dc}^2 c^2) \varphi_{dc} + (C_{dc}^1 b - C_{dc}^2 c) \dot{z}_{dc} + (k_{dc}^1 b - k_{dc}^2 c) z_{dc} = P_{\Sigma} c;$$

In which:

$m_{dc}$  – The volume of cluster engine

$z_{dc}$  – Transposition of front wheel, rear wheel and vehicle body

$C_{dc}^1, C_{dc}^2, k_{dc}^1, k_{dc}^2$  – Coefficient value and hardness coefficient value of front bearing and rear bearing

$J_{dc}$  – Inertia moment at centre of gravity of vehicle body

$P_{\Sigma}$  – General force acting on the engine body

a, b – The distance from the bearing to the center of gravity of the engine

c – The distance between the location of general force and coordinates the center of gravity of the engine

2. Construction method of solving system of differential equations

System of oscillating differential equation of the cluster engine and vehicle body has presented above can be written as:

In which:

$W(j\omega)$  – Frequency function ( $j = \sqrt{-1}$ ),

G – External forces acting column vector

$$M\ddot{x}_i + K\dot{x}_i + Cx_i = Q(t), \tag{3.1}$$

In which:

$x_i$  – Transpose column vector

M – Mass matrix

C – Hardness coefficient matrix

K – Coefficient of absorber matrix

Q(t) – External forces acting column vector

As we know, to solve system of oscillating differential equation, which has linear components, We use the Laplace transform as follows:

$$L \cdot [M\ddot{x}_i + K\dot{x}_i + Cx_i] = L[Q(t)], \tag{3.2}$$

$$\Rightarrow (-\omega^2 M + j\omega C + K) \cdot W(j\omega) = G,$$

$$\Rightarrow W(j\omega) = D^{-1} \cdot G, (D = -\omega^2 M + j\omega C + K).$$

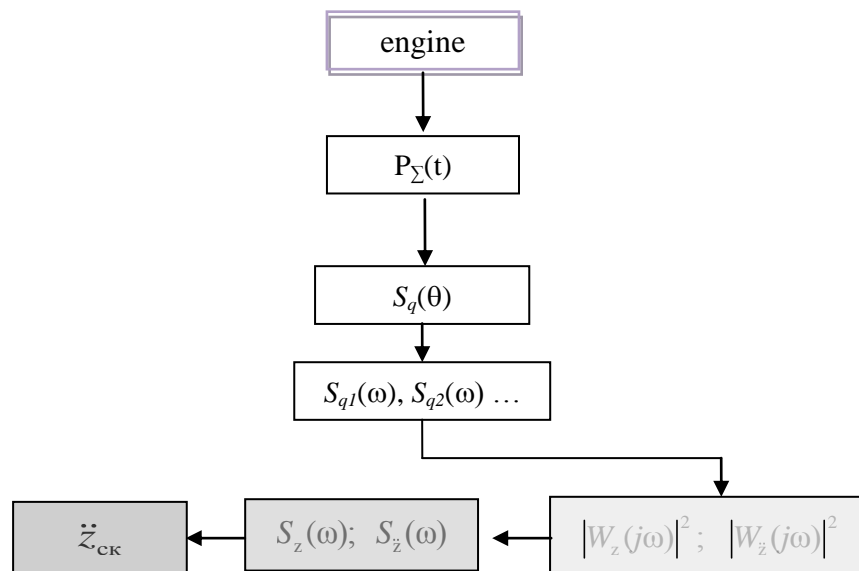


Figure 3: Calculation diagram of system of oscillating differential equation

Spectral density value vibrational transposition is calculated according to the formula:

$$S_z(\omega) = |W_z(j\omega)|^2 \cdot S_q(\omega), \tag{3.3}$$

In which:  $S_q(\omega)$  – Spectral density linear deformation

Spectral density value vibrational acceleration is calculated according to the formula:

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$$S_z(\omega) = |W_z(\omega)|^2 \cdot S_q(\omega) = \omega^4 |W_z(\omega)|^2 \cdot S_q \quad (3.4)$$

The average squared values of vibrational acceleration:

$$\ddot{z}_{ck} = \sqrt{\int_{-\infty}^{\infty} S_z(\omega) d\omega}, \quad (3.5)$$

**3. Computational results:**

The MATLAB software is used to find out the value of the frequency density spectrum, the average squared vibrational acceleration of engine cluster focus.

The results of the calculation of frequency density spectrum vibrational transposition and acceleration, expressed in the Figure 4, Figure 5, Figure 6 and Figure 7.

The results of the calculation of average squared transposition and acceleration, expressed in the Figure 8, Figure 9, Figure 10 and Figure 11.

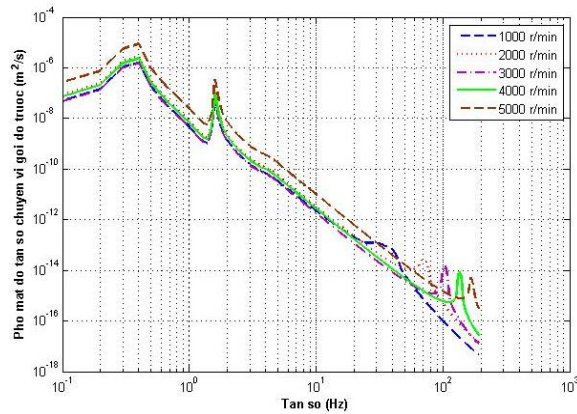


Figure 4. Frequency density spectrum vibrational transposition at the location of the front bearing

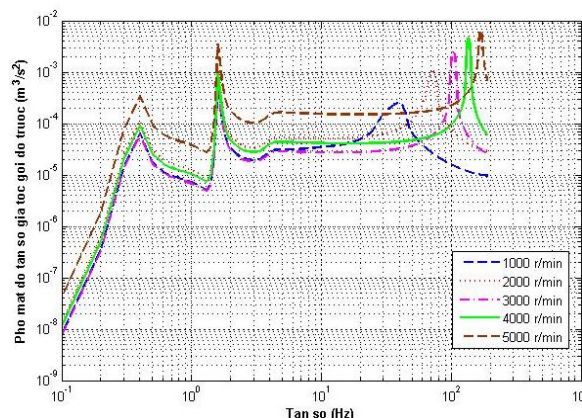


Figure 5. Frequency density spectrum vibrational acceleration at the location of the front bearing

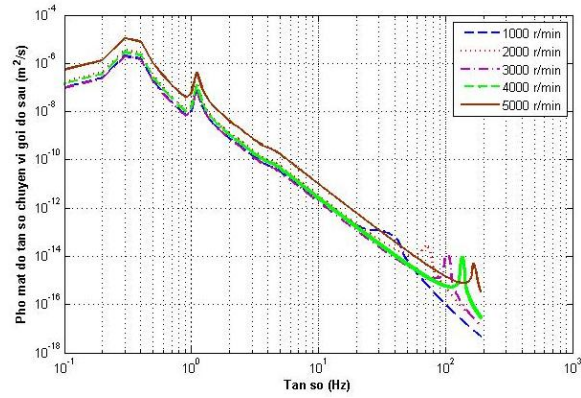


Figure 6. Frequency density spectrum vibrational transposition at the location of the rear bearing

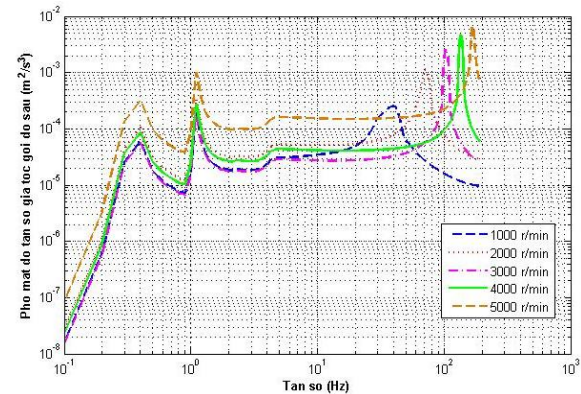


Figure 7. Frequency density spectrum vibrational acceleration at the location of the rear bearing

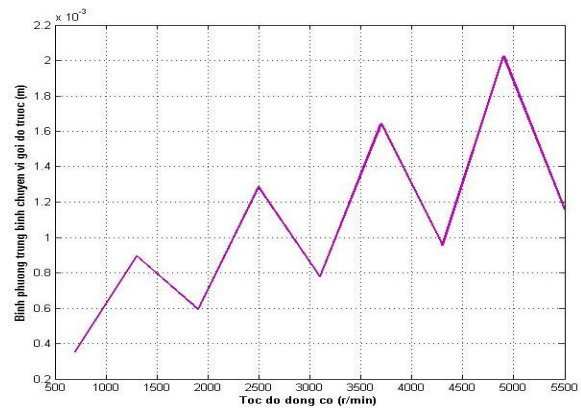


Figure 8. Average squared vibrational transposition at the location of the front bearing in different speeds

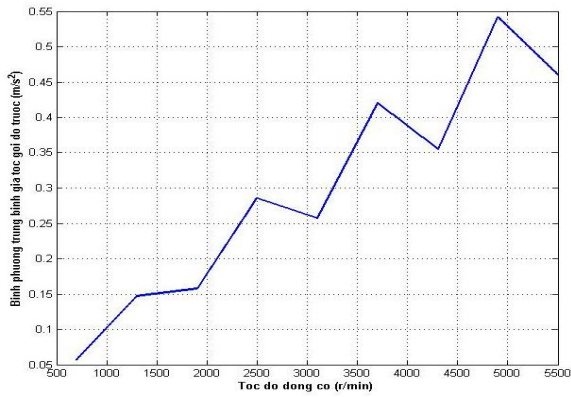


Figure 9. Average squared vibrational acceleration at the location of the front bearing in different speeds

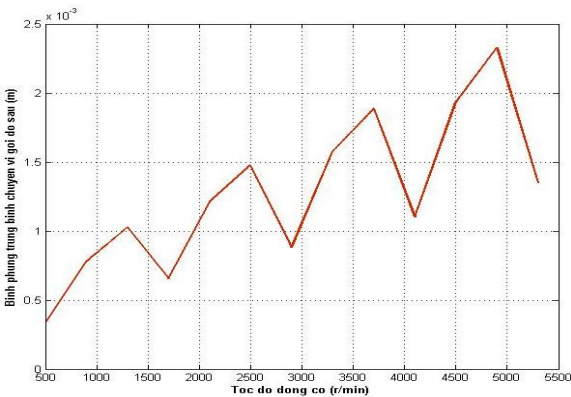


Figure 10. Average squared vibrational transposition at the location of the rear bearing in different speeds

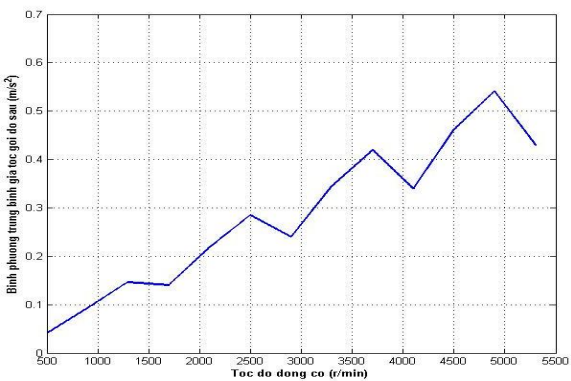


Figure 11. Average squared vibrational acceleration at the location of the rear bearing in different speeds

#### 4. Optimum bearing parameters by the graphical method

With the results of calculation of average squared vibrational transposition and acceleration at the location of the front and rear bearing according to the values of different speeds, we take the average and draw the graph of the average squared vibrational

transposition and acceleration according to the stiffness of the front and rear bearing as follows:

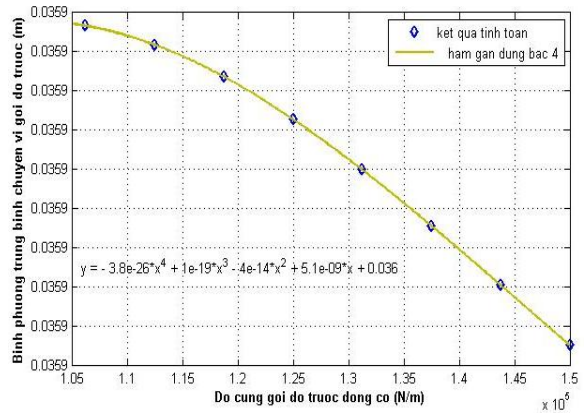


Figure 12. Average squared vibrational transposition at the location of the front bearing in different stiffness

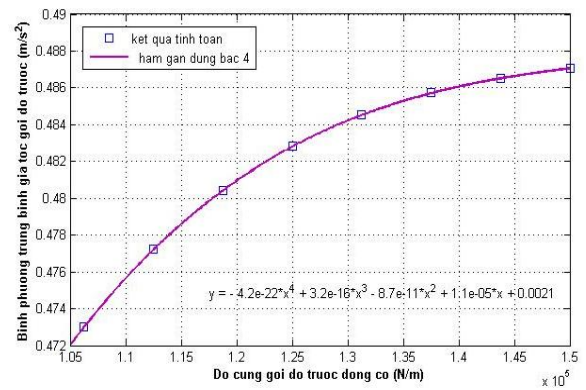


Figure 13. Average squared vibrational acceleration at the location of the front bearing in different stiffness

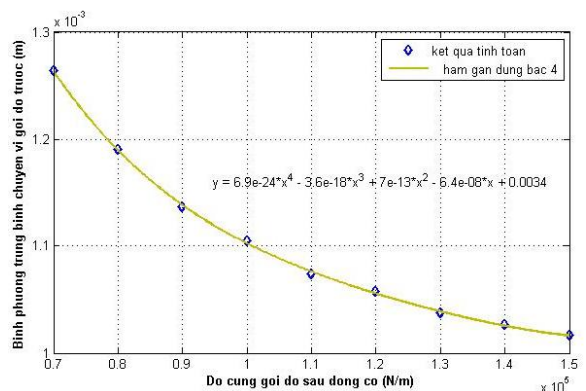


Figure 14. Average squared vibrational transposition at the location of the rear bearing in different stiffness

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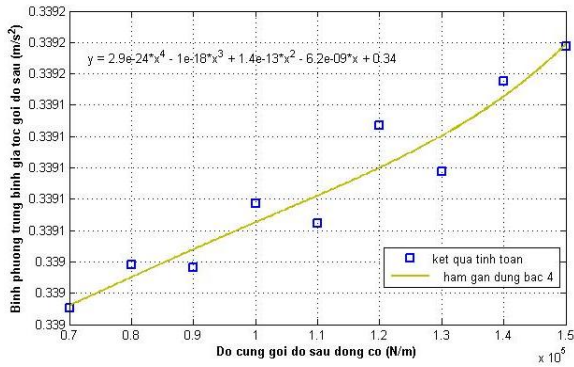


Figure 15. Average squared vibrational acceleration at the location of the rear bearing in different stiffness

The method to approximate a function is used to find out the approximate quartic function which expresses values of vibrational transposition and acceleration at the location of the front and rear bearing, depends on the different stiffness.

From the results of function which is obtained, we use the method of drawing stacked graph, obtained the results as on Figure 16 and 17

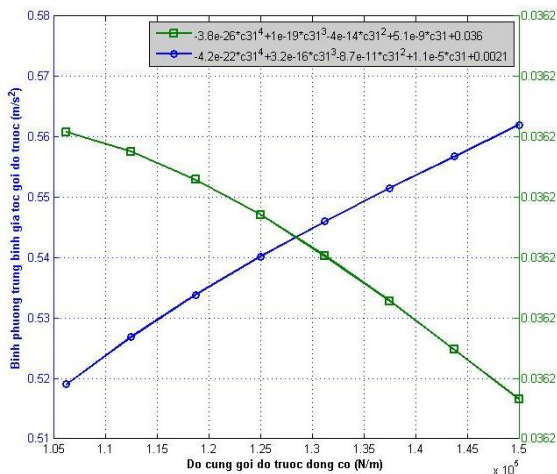


Figure 16. Stacked graph of average squared vibrational transposition and acceleration at the location of the front bearing

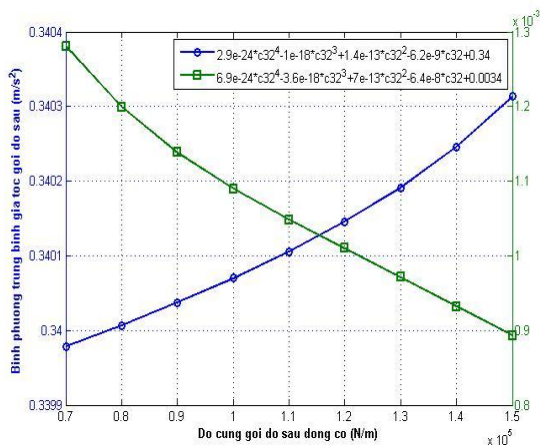


Figure 17. Stacked graph of average squared vibrational transposition and acceleration at the location of the rear bearing

Based on the function on the graph of Figure 16 and Figure 17, we find

- Optimal stiffness values of the front bearing in this case is: 128,405 N/m
- Optimal stiffness values of the rear bearing in this case is: 115,490 N/m

5. Conclusion:

- 1) The articles presents vibrational models and computational methods for clusters of engine, in which the results have shown the vibrational frequency spectral density and the average squared values of vibrational transposition and acceleration;
- 2) The article presents the graphical method to determine optimal stiffness values of of the front and rear bearing correctly;
- 3) Calculating the optimum bearing parameters by the graphical method above can be applied to solve similar optimization, for example: bearing of the type of motivation, bearing of the oscillating weight, stiffness of linear suspension cars ...

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