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Design optimization of cantilever retaining wall using direct optimal design simulation

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Abstract: Retaining walls are designed based on different approaches and most of the time they are oversized and are uneconomical to cast. Most of researchers have developed routine to optimize the design of retaining wall. In this research paper, study of different techniques is performed and concluded the simplest algorithm can analyze the cantilever wall with initial design input satisfying both internal and external constraints.

Keywords: *Retaining wall Optimization, Cantilever Wall, Flexural Design, Shear Design, Cost Function*

Introduction:

A number of parameters of retaining wall have been studied under the optimal design of retaining wall including geotechnical stability, wall topography, structural stability and optimized position on inclined hillside. Advanced studies based on Finite element analysis (FEA) were adopted to design the status of mechanics, Hou (2004). The optimized results presented that, with mild fill subgrade, stability of the wall was prominent, even notable deflection in the model were noted.

Several researchers have worked out on the formulation of the optimal algorithm for design of retaining wall, Street & Rhomberg (1981), Alshavi et al. (1981), Fang et al. (1980), Erbatur & Saribas (1996), Chi & Dembicki (1989), Pochtman et al. (1989) and Adidam & Keskar (1989).

The applied stresses on retaining wall was increased, and the calculated set of stresses were used for highfill subgrade. In cantilever-anchor model, anchor plate utilize in research is assumed as passive portion. This research can impart a significant change provided the wall has faced a prominent horizontal displacement. Hence, prestressed anchor cables and prestressed dispersed anchor plates must be applied so as to improve the stability of structure and minimize the horizontal displacement.

Failure possibilities of retaining wall are analysed by Zevgolis (2010). Structure stability was not enough for high fill subgrade. Cantilever retaining wall with mechanically stabilized supports was studied by Chen (2005).

Several sequences for base arrangement validate primary design checks for minimal factor of safety by sequel calculation for achieving structure dimension of minimized cost of the wall, (Street & Rhomberg, 1981).

Erbatur & Saribus (1996) have researched on reinforced concrete cantilever retaining wall optimization techniques, objective function for analysis was selected as cost and weight of the retaining wall. In the study, constraint for retaining wall were failure against overturning & sliding, minimal or no tension stress in the bas foundation, moment and shear capacity of retaining wall components.

Duncan (2000) & Whitman (2000 concluded in their research for requirement of probabilistic analysis in addition to factor of safety of retaining wall, because probabilistic procedure has direct linkage to probability of structure failure & design parameter uncertainity.

Design Constraints and Methodology:

The optimized design of retaining wall, including stress constraints, is formulated below. The design problem is encoded in Visual Basic .Net programming language. Calculate the cross sectional dimensions, including toe width (b_t), toe thickness (t_{ts}), heel width (b_h), heel thickness (t_{hs}), top thickness of vertical stem (b_{at}), inner face inclination of stem (α_1), outer face inclination of stem (α_2); base key width (b_k), key depth (h_k) and its location w.r.t heel (b_{kh}); reinforcement in toe (A_{st}, A_{sdt}), heel (A_{sh}, A_{sdh}), stem (A_{sad}, A_{sa23}, A_{sa13}) and key (A_{sk}, A_{sdk}) to withstand the retained soil safely and accomplish minimal cost in the same time amongst the all feasible calculated designs. Mathematically, the design problem is formulated as follows. Design vector is presented as

$$X = x_i = (t_{ts}, t_{hs}, h_k, b_t, b_h, b_{at}, b_k, \alpha_1, \alpha_2)$$
$$i = 1, 2, \ldots, N$$

Such that the objective function

$$F_c = \sum_{i=1}^{M} UR_i \times Q_i$$

achieve the minimal value among the all calculated feasible geometries, satisfying the explicit constraints.

$$h^U \geq h \geq h^L$$
$$b^U \geq b \geq b^L$$
$$\alpha^U \geq \alpha \geq \alpha^L$$

and M implicit constraints.

$$g_j(x_i) \leq 0$$
$$j = 1, 2, \ldots, M$$

where x_i represent design vector, N is the dimensions of design space, M represents number of implicit constraints, UR_i and Q_i are the unit rate and quantity of excavation, backfilling, concrete, reinforcement and formwork respectively. Superscripts U and L denote upper and lower boundaries on design variable.

Implicit constraints impose restriction on stresses as governed by ACI specifications.
Variable design vector is represented in the first equation. In practice, not all elements of the design vector are independent variables of the design space. Some of the variables may be linked in order to satisfy symmetry. Thus, the dimension of design vector in a particular case may be smaller than what has been suggested for more general case.

Solution Procedure:
To solve the stated optimization problem, a computational methodology is encoded consisting of three logically separable phases: the optimization phase, the structural analysis phase, and the design evaluation phase. During the optimization phase, attempts are made to improve by finding feasible points that are successively closer to an optimum. In the structural analysis phase, the structure, provisionally obtained in the optimization phase, is analyzed and, finally, the feasibility of the structure is checked in the design evaluation phase. An overview of the Complex Method of Box is given below.

Complex Method of Box:
The Complex Method is a mathematical programming procedure for finding an optimal solution of non-linear, constrained optimization problems. This method derives its acronym COMPLEX from two words, Constrained and Simplex. The Complex method was proposed originally by M. J. Box in 1965, where he demonstrated efficacy of the method in finding near optimal solution to non-linear, constrained optimization problems. It is a Zero-order method optimization method; that is, it does not require either the gradient of objective function, or that of constraints. The choice of Complex Method was made for its ability to span large portions of the design space , thereby providing a better chance of finding the global optimum, and for its ability to deal with constrained optimization problems.
The method attempt to find a design vector

$$x = x_i \qquad i = 1, 2, \ldots, N$$

To minimize the function

$$f(x_i) \qquad i = 1, 2, \ldots, N$$

Subject to N explicit constraints

$$x_i{}^U \geq x_i \geq x_i{}^L \quad i = 1, 2, \ldots, N$$

And M implicit constraints

$$g_j(x_i) \leq 0 \;\; j = 1,2, \ldots M \;\; i = 1, 2, \ldots, N$$

Where

N = Number of Design Variables
M = Number of Implicit Constraints
x_i^L = Lower Bound Limit for design variables
x_i^U = Upper Bound Limit for design variables

The Complex Method optimizes a provisional design by reflecting the worst point (design) through the centroid to find the best point (design). The optimization process is divided into two phases. In the first phase a set of feasible points (satisfying all constraints) are generated randomly. After generating the initial complex, the algorithm moves to the reflection phase. In this phase, the method calls for the improvement of the worst point in the complex. To improve a point, the algorithm reflects it through the centroid of the remaining points (vertices of the Complex). If the reflected point is worst, or it violates an implicit constraint, it is moved back half the distance to the centroid. The method continues in this manner until convergence criteria are met, or the maximum number of iterations is reached. Details of the method and its successful application to structural design problems can be found in references

Modification and Complex method: Implementation:
The modifications to the complex Method as used in this study are summarized as follows.

a. The Improvement Procedure:
The improvement procedure has been modified in that at every iteration the worst design is reflected through the centroid of remaining designs in the design space to a new point. Then, when this new point has been optimally sized, its objective function is evaluated and compared with that of worst design in the complex. If the new point is less, it is accepted as a design improvement and termination criteria are checked; if greater, instead of continuously halving α, it is halved only thrice and then centroid is considered as a candidate for improvement. If centroid is still greater than the worst, then a new point is located at the midpoint of a line joining centroid to the best point in the complex. If the objective function is still greater than the worst, then the worst point is replaced by the best design in the complex.

b. Termination Criteria:
The procedure is repeated until a preset termination criterion is reached. The first termination criterion used in this study is based on the objective function values of all point in the complex. This convergence criterion is met if the ratio of the difference between the maximum objective function value and the minimum objective function value to maximum objective function value of the points in the complex is less than or equal to the value of (a user define variable) i.e.

$$\epsilon \geq \frac{(f_{max} - f_{min})}{f_{max}}$$

The second criterion that is checked for the convergence of the solution is a measure of the design space spanned by the vertices of the complex,

$$\delta \geq \frac{\sum_{i=1}^{n}\sum_{j=1}^{k}\left(x_{ij} - c_i\right)^2}{2\sum_{i=1}^{n}(x_i{}^U - x_i{}^L)^2}$$

Where $$c_i = \frac{1}{k}\sum_{j=1}^{k} x_{ij}$$

Finally, a constraint is placed on the maximum number of iterations that may occur before terminating the optimization. The optimization process is terminated as soon as any of the termination criteria is satisfied.

Computational Examples:

Two numerical examples are solved to demonstrate the versatility of the proposed procedure. The results of the examples were generated with Visual basic 2012 Express edition program executed on Core(TM) i3, 2.20 GHz laptop computer with 2.00 GB of RAM. This program is in three interacting modules, performs search for optimum, structural analysis and structural design. In the development of optimization routine guidance is taken from Muhammad Rizwan (et.al, Bashir (et.al) in order to produce modified version of complex method. The structural analysis and structural design routine is developed by using Design of Concrete Structures by Arthur H. Nilson, 13th edition. The decision-making and the computational assignments are carried out by separate subroutine in each phase. This feature is highly desirable, especially when the need for modification to the design code may arise in the future. However, different modules representing the provision of other commonly used design codes can be appended to the existing design routine with relative ease.

The material specified for retaining wall is concrete with crushing strength of 20 kN/sq m (normal- weight concrete), flexural steel with ultimate strength of 415 kN/sq m. The implicit constraints are to meet the relevant provisions of ACI specification.

1. Example 01:

The objective of this design exercise is to reduce the construction cost of the cantilever wall to retain the soil load of depth 4.7 m with unit weight of 15 kN/cu m and angle of internal coefficient of friction as 28 degrees, the cross sectional dimension of the elements are presented in figure 1 including t_{ts}, b_t, t_{hs}, b_h, b_{at}, α_1, α_2, h_k, b_k and b_{kh} as thickness and width of toe, thickness and width of heel, thickness and inclinations of vertical stem; depth and width of base key and its location w.r.t. heel, respectively, so that wall withstand retained soil fulfilling ACI stress requirements. The histories of the constraints represent the wall capacity vs soil demand graphs in figure 2 verifying the non-violation condition of the constraints.

The software accomplished the result within 192 iterations. Initially 20 randomly designs are generated in the complex ranging from 1.35 M. Rs. to 1.80 M. Rs.

The wall at iteration 1 and 116 is 16.83% and 6.16% more expensive than the optimized cost of wall. Maximum and minimum cost function is plotted against successive iterations is plotted in figure 2. The optimized cost calculated as per ACI specifications for this example is Rs. 1.16 million.

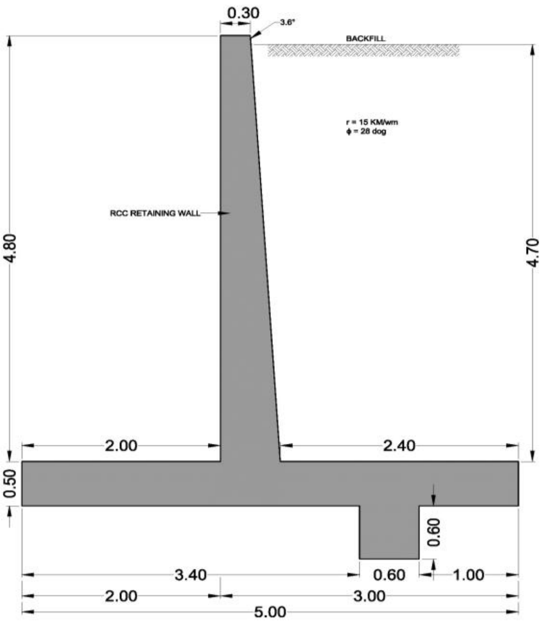

Figure 1: *Geometric Dimensions of the Retaining Wall.*

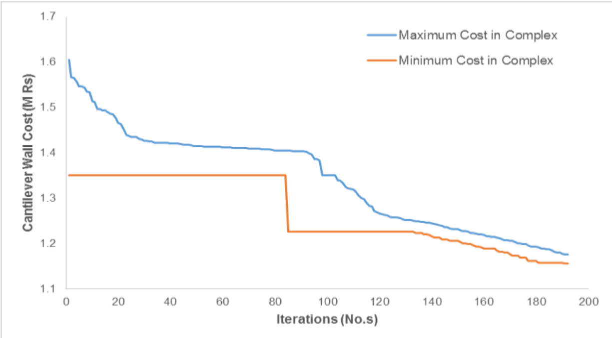

Figure 2: *Design Complex maximum and minimum cost analysis*

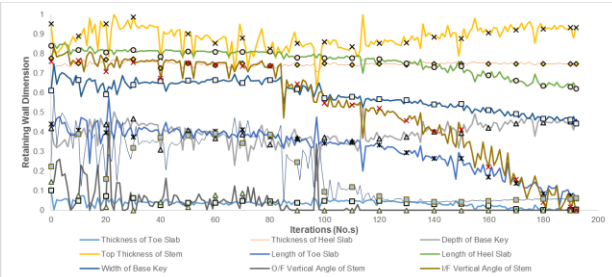

Figure 3: *History of design constraints*

2. Example 02:

Consider a 5.5 m high retaining wall need to support 5 m deep soil having unit weight of 16 kN/cu m and angle of internal friction as 30 deg. Geometric dimension of the wall is presented in figure 4. The history of the constraints is plotted in figure 5 clearly explaining that the constraints have not violated the limits.

Computation of retaining wall design required 113 numbers of iterations. The economical cost in the complex is computed as 1.13 M Rs. At the iterations 1 and 35 the wall is 13.69% and 5.57% respectively costly than the optimized design. The maximum and minimum cost function of the wall is plotted against the iterations in figure 5.

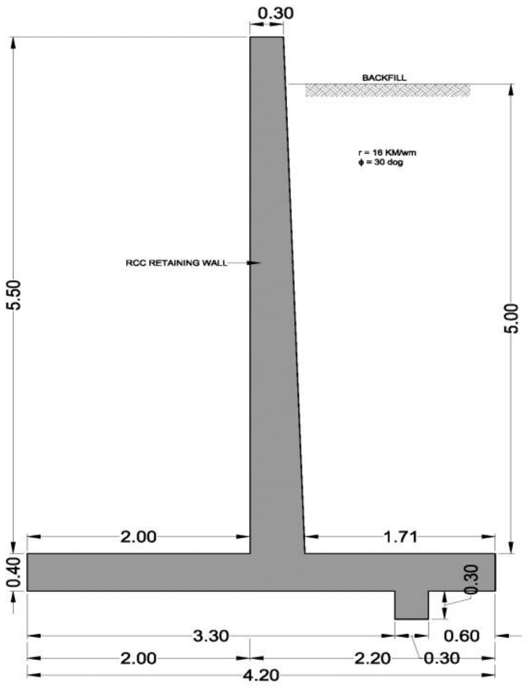

Figure4: *Geometric Dimensions of the Retaining Wall.*

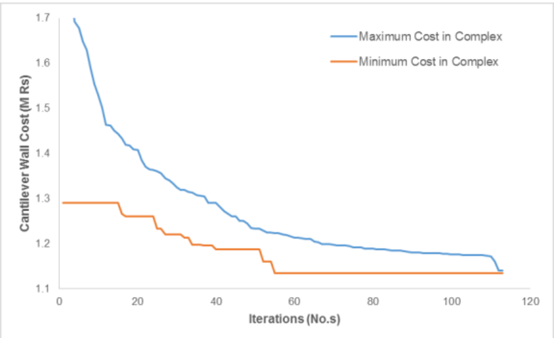

Figure5: *Design Complex maximum and minimum cost analysis*

Conclusions:

Direct optimization technique on the basis of Complex Method with justifiable alterations analysed in this paper is clear to be utilized for design of cantilever retaining wall. The objective function analyse the structural dimensions as external constraint for calculating member strength. The design module has capability to explore design vertices of the provided model and this algorithm is effective in computation of optimized design cost of cantilever retaining wall. The module requires initial point satisfying internal constraints, later on design vertices are generated and computed for improvement in each element of the structure. This routine improves the power of calculating global optimized result of the problem.

Time taken and amount of calculation for optimal design procedure depends upon selection of number of generated points in the design space and reflection factor α (in this report importance factor used is 1.3 as suggested by Box). Designer can edit these computational parameters for improvement in the optimal problems.

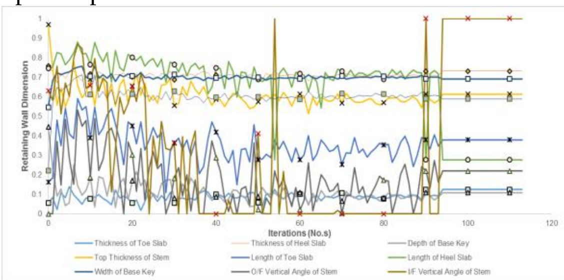

Figure6: *History of design variables*

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